Domain-Independent Dynamic Programming

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Research Question
Can we use dynamic programming as a model-based paradigm for combinatorial optimization?

Developed Software
Use our software to solve your problem by just defining a DP model!
We developed general-purpose heuristic search solvers!
Install the Python interface: pip install didppy
Open-source and free for commercial use (MIT/Apache-2.0).

TSPTW Example
Minimize the travel time of a tour to visit all customers within the time windows.

DP Model for TSPTW
\[ V(N \setminus \{0\}, 0, 0) = \min_{\sum j \in U \setminus \{0\}, c_{ij} \leq b_j} \left( \begin{array}{c} c_{ij} + V(U \setminus \{j\}, j, \max\{t + c_{ij}, a_j\}) \text{ if } U \neq \emptyset \\ \alpha_0 + V(\emptyset, 0, 0 + t + c_{ao}) \\ 0 \end{array} \right) \]

State variables:
- \( U \): unvisited customers
- \( i \): current customer
- \( t \): current time

Constants
- \( N \): all customers (0: depot)
- \( a_i, b_i \): time window for customer \( i \)
- \( c_{ij} \): travel time from customer \( i \) to \( j \)

What DIDP Can Do but PDDL Cannot
Explicitly modeling implications of the problem definition that can be useful solvers (common in OR!).
State constraints (can be used for pruning)
\[ V(U, i, t) = \infty \text{ if } \exists j \in U, t + c_{ij} > b_j \]

Dominance with resource variables (can be used for pruning)
\[ V(U, i, t) \leq V(U, i, t') \text{ if } t' \leq t \]

Dual bound (can be used as a heuristic)
\[ V(U, i, t) \geq 0 \]

Modeling and Solving in DIDP

Heuristic Search Solvers
Heuristic search solves a DP model as a shortest path problem in a state space using the dual bound as a heuristic.

We developed the following solvers:
- CAASDy: A*.
- CABS: performs beam search with exponentially increasing beam width (anytime and complete).
- 5 other anytime heuristic search solvers.

Promising performance compared to MIP and CP.

Future work: parallelization, domain-independent dual bound.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
<th>MIP</th>
<th>CP</th>
<th>CAASDy</th>
<th>CABS</th>
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</thead>
<tbody>
<tr>
<td>TSPTW (340)</td>
<td>TSP with time</td>
<td>227/0.227</td>
<td>47/0.026</td>
<td>257/0.244</td>
<td>259/0.003</td>
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<tr>
<td>CVRP (207)</td>
<td>Vehicle routing</td>
<td>26/0.585</td>
<td>0/0.317</td>
<td>50/0.976</td>
<td>6/0.185</td>
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<tr>
<td>SALBP-1 (2100)</td>
<td>Assembly line</td>
<td>1357/0.345</td>
<td>1584/0.005</td>
<td>1653/0.213</td>
<td>1801/0.000</td>
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<tr>
<td>Bin Packing (1615)</td>
<td>Bin packing</td>
<td>1157/0.039</td>
<td>1234/0.002</td>
<td>922/0.429</td>
<td>1163/0.002</td>
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<tr>
<td>MOSP (570)</td>
<td>Manufacturing</td>
<td>225/0.039</td>
<td>437/0.004</td>
<td>483/0.153</td>
<td>527/0.000</td>
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<tr>
<td>Graph-Clear (135)</td>
<td>Building security</td>
<td>24/0.110</td>
<td>4/0.015</td>
<td>760/4.357</td>
<td>1030/0.000</td>
</tr>
<tr>
<td>Talent Scheduling (1000)</td>
<td>Scheduling actors</td>
<td>6/0.051</td>
<td>7/0.002</td>
<td>224/0.793</td>
<td>253/0.011</td>
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<tr>
<td>m-PDTS (1117)</td>
<td>Pick up &amp; delivery</td>
<td>945/0.078</td>
<td>1049/0.013</td>
<td>947/0.196</td>
<td>1035/0.002</td>
</tr>
<tr>
<td>1/\sum_{i,j} T_i (375)</td>
<td>Job scheduling</td>
<td>109/0.018</td>
<td>15/0.000</td>
<td>270/0.280</td>
<td>285/0.034</td>
</tr>
</tbody>
</table>

Coverage / primal gap (gap to the best known cost) achieved within 8GB and 30-min.